State Dependent Preferences and the Equity Premium Puzzle: A different Perspective

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Abstract

This paper revisits state dependent recursive preferences in addressing the equity premium puzzle. The model's calibration generates countercyclical relative risk aversion and elasticity of intertemporal substitution. The Stochastic discount factor varies significantly across states and satisfies Hansen-Jagannathan bound. Further, I find a mixed risk averse representative agent best explains data. The aforementioned results are robust to changing the developed country being investigated and the subjective discount factor value.

Keywords: State dependent preferences, Epstein-Zin preferences, equity premium, Risk aversion, Elasticity of intertemporal substitution.

JEL Classification: G10, G12.

"If at first the idea is not absurd, then there is no hope for it" ~ Albert Einstein

1. Introduction

An unresolved question yet: What can explain the high levels of historical equity premium? Why theoretical models' representative agent has high relative risk aversion, higher than acceptable, to explain equity premium levels? The goal of this paper is to re-examine the special class of recursive preferences in

addressing the equity premium puzzle while allowing preferences to be state dependent. I calibrate the model developed in Melino and Yang (2003) to the same sample studied in their paper but results support the idea of a mixed risk averse representative agent. This type of agent behaves as risk averse in some states while he behaves as a risk lover in others. One contribution is introducing, for the first time in the equity premium puzzle literature, the concept of mixed risk aversion. Another contribution of the paper is examining the results for selected developed countries other than the United States as well as extending the sample of the United States to include recent observations until 2012. All results support that a mixed risk averse representative agent better matches historical asset market characteristics. The model's stochastic discount factor variability is high and satisfying Hansen-Jagannathan bound[1]. The model resulted in countercyclical relative risk aversion and elasticity of intertemporal substitution coefficients. Empirical results suggest low values for the elasticity of intertemporal substitution, i.e. close to zero.

The advantage of recursive preferences is the desirable characteristic of disentangling risk aversion and elasticity of intertemporal substitution coefficients. In addition, they satisfy early resolution preference by agents. Allowing both the relative risk aversion coefficient and the elasticity of intertemporal substitution parameters to be state dependent generates a stochastic discount factor which varies across states, depending on both current and upcoming states of nature. It has been emphasized throughout the literature the need of high variability in stochastic discount factor values to explain the equity premium puzzle.

In Mehra and Prescott (1985) model with isoelastic preferences the stochastic discount factor varies over time only with the rate of consumption growth realized next time period. While state independent recursive preferences' stochastic discount factor, as in Epstein and Zin (1989 and 2001) and Weil (1990), is more flexible as it depends on both states; the current and the upcoming one. With state dependent preferences, not only the stochastic discount factor but also the risk aversion and elasticity of intertemporal substitution parameters vary across states, adding more flexibility to the model.

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[1] That is the ratio of the standard deviation of the stochastic discount factor to its mean should be at least equal to the sharp ratio attained by any portfolio.

Throughout the literature, countercyclical risk aversion has been proposed as an explanation for the equity premium puzzle. However, in Melino and Yang (2003), countercyclical risk aversion played an important role in explaining the equity premium puzzle only when combined with modest cyclical variation in intertemporal substitution. Their results support the idea of allowing both relative risk aversion and elasticity of intertemporal substitution to vary across states contemporaneously.

Countercyclical relative risk aversion is intuitive because when the risk averse representative agent experiences a low draw for consumption growth, he is expected to assign a high value to assets which pay off units of consumption; given the next period's state is also a low one. At the same time, he is expected to assign a relatively low value to assets which pay off units of consumption if the high state will realize next period. Informally, the representative agent is expected to act as if he is very risk averse during recessions, but his risk aversion should be lower during booms. This suggests a different behavior for the representative agent across states. With state dependent preferences, the difference in the agent's attitude across states is more obvious and tractable, if there is any.

On the other hand, elasticity of intertemporal substitution plays a key role in macroeconomic analysis as well as in finance. In Epstein-Zin preferences, the elasticity of intertemporal substitution is the key parameter in optimal consumption rule; see Campbell and Viceira (1999). Most of the literature assumes the elasticity of intertemporal substitution is constant. Few exceptions explored whether elasticity of intertemporal substitution varies with consumption's (or wealth's) level or it remains constant, such as Blundell, Browning and Meghir (1994), Atkeson and Ogaki (1996), and Attanasio and Browning (1995). They could not accept the constant elasticity of intertemporal substitution hypothesis; however, these rejections tend to be statistically insignificant.

Crossley and Low (2011) declared the assumption of a constant elasticity of intertemporal substitution is strong. Likewise, Georgiev (2004) emphasized the variability of elasticity of intertemporal substitution remains important if one is to rely only on consumption growth variation to match returns' volatility across states.

Hence, it seems reasonable to address state dependent risk aversion as well as elasticity of intertemporal substitution coefficients. It is worth to mention, Melino and Yang have studied the state dependent preferences previously in 2003. Nevertheless, I would like to explore the same preferences, approaching the solution differently. The aforementioned points together motivated myself to study state dependent recursive preferences' effects on explaining equity premium data.

"*Mehra and Prescott (1985) show that the difference in the covariances of these returns with consumption growth is only large enough to explain the difference in the average returns if the typical investor is implausibly averse to risk. This is the Equity Premium Puzzle: in a quantitative sense, stocks are not sufficiently riskier than Treasury bills to explain the spread in their returns.*" – Kocherlakota 1996, *The equity Premium: It's still a Puzzle*.

Among other approaches[2], many economists have attempted to solve the puzzle by introducing more general preferences. Modifying preferences is a response to conventional consumption asset pricing model's critics. It has been criticized for two main reasons. Firstly, it does not perform well empirically; one example is the equity premium puzzle. Secondly it restricts the risk aversion and intertemporal substitution parameter to be inversely related to each other by using standard isoelastic preferences. Various preference orderings have appeared in the literature, typically by introducing additional state variables.

The long standing paradox of conventional macro-economic asset pricing models motivated introducing state dependence to preferences in the C-CAPM. I elaborate on the latest contributions to state dependent preferences. Gordon and St. Amour (2000) developed a consumption based asset pricing model in which attitudes towards risk are contingent upon the state of the world. It can be considered as an unrestricted approach to time varying prices of risk in which the state variable underlying the representative agent's preferences is a latent variable. They found, when risk aversion is state dependent, low (high) consumption and countercyclical (pro-cyclical) curvature implies more variation in the marginal utility. Their two state

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[2] Among the surveys which reviewed the equity premium puzzle literature: Abel (1991), Cochrane and Hansen (1992), Heaton and Lucas (1995), Kocherlakota (1996), Cochrane (1997), Campbell (1999 and 2001) and Mehra and Prescott (2003).

Markov preferences model could match the first two moments of returns and bonds as well as the cyclical nature of equity prices.

In 2004 they considered again risk aversion as a latent variable but in an unrestricted reduced form with a large number of homogenous agents. Using Bayesian Markov chain Monte Carlo technique, their model could justify high observed premia on risky assets in the United States with plausible levels of risk aversion.

In both studies they considered isoelastic preferences. Even though they could provide more plausible results, the isoelastic utility model is still limited. Isoelastic preferences restrict risk aversion to be inversely related to elasticity of intertemporal substitution when there is a priori reason it must be so.

On the contrary, recursive preferences, e.g. Epstein and Zin (1989) and Weil (1990), can parameterize both the elasticity of intertemporal substitution and the coefficient of relative risk aversion independently, which constitutes an additional degree of freedom to replicate the level of risk free rate and risk premium. Accordingly, the risk free rate is mainly controlled by the magnitude of the elasticity of intertemporal substitution, while the risk premium is a reflection of the coefficient of relative risk aversion. Moreover, recursive preferences satisfy early resolution preference by agents.

Melino and Yang (2003) applied state dependence to recursive preferences in a Mehra-Prescott economy. They found countercyclical risk aversion plays an important role in explaining the equity premium puzzle, as is emphasized in the literature, but only when combined with modest cyclical variation in intertemporal substitution.

Equivalent to Melino and Yang (2003), I approach the puzzle using their theoretical model of state dependent recursive preferences in a Mehra-Prescott economy. I would like to explore the research questions: Can state dependent recursive preferences provide a better explanation for equity premium high levels? How different states of nature might affect the behavior of the representative agent? Knowing how different the economic agent's response to different states of nature is essential. Its relevance relies in being able to form a better understanding of his reactions and what it might lead to. For example, financial intermediaries are required by the European Union Directive on Financial instruments (MiFID) to design the

best strategy to elicit the degree of risk aversion of their customers. This shows the importance of understanding agents' behavior.

Using historical annual data for selected developed countries, the model's representative agent is of a mixed risk aversion type. This result leads to a stochastic discount factor which varies significantly across states as well as satisfying Hansen-Jagannathan bound. Besides, both relative risk aversion and elasticity of intertemporal substitution are countercyclical.

2. Theoretical Framework

The model considered in this paper was introduced in Melino and Yang (2003), to which the reader is referred for a complete analysis. In this section, I highlight the major features of their model; preferences, budget constraint and equilibrium.

2.1 The Model

Building on the literature of temporal Von Neumann Morgenstern recursive preferences[3]; in other words Epstein-Zin preferences, Melino and Yang (2003) set up this model. They consider an infinitely-lived representative agent, who receives utility from the consumption of a single good in each period. The representative agent has recursive preferences as the following:

$$U_t = \left(c_t^{\rho(s_t)} + \beta \left[E_t \left(U_{t+1}^{\alpha(s_t)} \right) \right]^{\frac{\rho(s_t)}{\alpha(s_t)}} \right)^{\frac{1}{\rho(s_t)}} \qquad (2.1)$$

With a certainty equivalence function $\mu_t = \left(E_t \left(U_{t+1}^{\alpha(s_t)} \right) \right)^{\frac{1}{\alpha(s_t)}}$

where E_t is the expectation conditional on period t information, c_t refers to the representative agent's real current consumption, β represents the agent's subjective discount factor, $\alpha(s_t)$ indicates the level of risk aversion of the representative agent, while $\rho(s_t)$ indicates the time preference of the agent. More precisely, the coefficient of relative risk aversion for 'timeless gambles' is $1 - \alpha(s_t)$; see Epstein and Zin

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[3] For further details and arguments about using recursive preferences, please refer to Kreps and Porteus (1979) as well as Epstein and Zin (1989).

(1989). While the elasticity of intertemporal substitution equals $1/(1 - \rho(s_t))$. $\alpha(s_t)$ and $\rho(s_t)$ depend on an exogenous state variable s_t.

I use Mehra-Prescott environment. It usually refers to the environment introduced in Mehra and Prescott (1985); a Lucas endowment economy in which real per capita consumption growth is a Markov stationary process. The economy is frictionless. At time t, current consumption is non-stochastic but future consumption levels are generally stochastic. There are two assets, one risky and another risk free. Equity as a stock market index represents the risky asset. The equity share is completely traded. Government short-term treasury bills represent the riskless asset. Under the Consumption Based Capital Asset Pricing Model CCAPM, the only state variable is real per capita consumption growth rate. I assume the subjective discount factor β is constant. I allow both α and ρ, or in other words the relative risk aversion and the elasticity of intertemporal substitution coefficients, to be state dependent.

States of nature are two; a high and a low one. I employ a simple threshold model to define the threshold between both states. I assume both states of nature are equally likely to happen with a symmetric transition matrix for the conditional probabilities.

$$\prod = \begin{bmatrix} \pi_{ll} & \pi_{lh} \\ \pi_{hl} & \pi_{hh} \end{bmatrix}$$

Where l and h refer to the low and the high state respectively. π_{ij} is the probability of going from state i to state j.

2.2 Equilibrium:

The representative agent maximizes his utility U_t subject to his wealth accumulation constraint

$$\max_{c_t} U_t = \left(c_t^{\rho(s_t)} + \beta \left[E_t \left(U_{t+1}^{\alpha(s_t)} \right) \right]^{\frac{\rho(s_t)}{\alpha(s_t)}} \right)^{\frac{1}{\rho(s_t)}}$$

$$s.t. \qquad x_{t+1} = (x_t - c_t) r_{t+1}$$

(2.2)

Where x_t is the beginning-of-period wealth, x_{t+1} is the representative agent's wealth at time $t + 1$ and r_{t+1} is the gross real return of the stock market index.

Equity is the only asset in non-zero net supply. In calculating the stock market index gross returns, whenever data on dividends are available, I use the formula:

$$r_{t+1} = \frac{p_{t+1} + d_{t+1}}{p_t} \qquad (2.3)$$

Where p_t is the stock market index's price at time t and d_{t+1} refers to the index's dividends at time $t+1$.

Otherwise, I utilize the total return index to calculate returns. In this case the market index gross returns equal to:

$$r_{t+1} = \frac{RI_{t+1}}{RI_t} \qquad (2.4)$$

Where RI_t is the value of the total return index at time t.

This maximization problem yields the following Euler equations as first order conditions:

$$E_t \left(\beta g_{t+1}^{\alpha(s_t)-1} \left(\frac{\beta M_{t,t+1}}{g_{t+1}} \right)^{\frac{\alpha(s_t)}{\rho(s_t)}-1} a_{t+1}^{\frac{\alpha(s_t)}{\rho(s_t)} - \frac{\alpha(s_t)}{\rho(s_{t+1})}} M_{t,t+1} \right) = 1 \qquad (2.6)$$

$$E_t \left(\beta g_{t+1}^{\alpha(s_t)-1} \left(\frac{\beta M_{t,t+1}}{g_{t+1}} \right)^{\frac{\alpha(s_t)}{\rho(s_t)}-1} a_{t+1}^{\frac{\alpha(s_t)}{\rho(s_t)} - \frac{\alpha(s_t)}{\rho(s_{t+1})}} rf_{t+1} \right) = 1 \qquad (2.7)$$

Where $M_{t,t+1} = r_{t+1}$ is the gross return of the stock market index from time t to $t+1$. $a_t \equiv c_t/x_t$ is the consumption wealth ratio and g_{t+1} refers to the real consumption growth rate at time $t+1$. rf_{t+1} indicates the gross return of the risk free asset.

Hence the Stochastic Discount Factor, or in other words the intertemporal marginal rate of substitution[4], formulation is:[5]

$$SDF_{t,t+1} = \beta g_{t+1}^{\alpha(s_t)-1} \left(\frac{\beta M_{t,t+1}}{g_{t+1}} \right)^{\frac{\alpha(s_t)}{\rho(s_t)}-1} a_{t+1}^{\frac{\alpha(s_t)}{\rho(s_t)} - \frac{\alpha(s_t)}{\rho(s_{t+1})}} \qquad (2.5)$$

At equilibrium, the stochastic discount factor equation can be simplified more. The budget constraint of the maximization problem (eq. 2.2) implies:

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[4] It is worthy to mention that the terms; stochastic discount factor and intertemporal rate of substitution, are used interchangeably.
[5] Melino and Yang (2003) provided a proof of equation 2.5.

$$x_{t+1} = (x_t - c_t)M_{t,t+1} = (1 - a_t)M_{t,t+1}x_t$$

Therefore,

$$a_{t+1} = {}^{c_{t+1}}\!/_{x_{t+1}} = {}^{g_{t+1}a_t}\!/_{[(1 - a_t)M_{t,t+1}]} \tag{2.8}$$

At equilibrium $c_{t+1} = d_{t+1}$ so the market portfolio is:

$$M_{t,t+1} = \frac{p_{t+1} + c_{t+1}}{p_t} \tag{2.9}$$

Where p_t is the price of the stock market index at time t and d_{t+1} is the index's dividend at time $t+1$.

Following Melino and Yang (2003), equilibria are considered where ex-dividend price of equity $p(g_t, c_t)$ is described by the time invariant and positive function of the variables g_t and c_t. Due to homogeneity of preferences, the price is linearly homogeneous in consumption. So $p(g, c) = p\left(\frac{g}{c}, 1\right) * c = PD(g) * c$ where $PD(g)$ is the price-dividend ratio.

Hence the stock market index's return can be written as:

$$M_{t,t+1} = \frac{PD(g_{t+1}) + 1}{PD(g_t)} g_{t+1} \tag{2.10}$$

In Lucas endowment equilibrium, the agent's wealth is:

$$x_t = p_t + c_t \tag{2.11}$$

Therefore the consumption-wealth ratio at equilibrium can be written as:

$$a_t \equiv \frac{c_t}{x_t} = \frac{1}{1 + {}^{p_t}\!/_{c_t}} = \frac{1}{1 + PD_t} \tag{2.12}$$

Substituting equations 2.10 and 2.12 in equation 2.5, the Stochastic Discount Factor at equilibrium can be rewritten equivalently as in equation 2.13 below.

$$SDF_{t,t+1} = \beta g_{t+1}^{\alpha(s_t)-1} \left(\frac{\beta}{PD_t}\right)^{\frac{\alpha(s_t)}{\rho(s_t)}-1} (1 + PD_{t+1})^{\frac{\alpha(s_t)}{\rho(s_{t+1})}-1} \tag{2.13}$$

3. Methodology and Data

In section three, I illustrate both methodology and data.

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3.1 Methodology

I calculate mathematically two values for real consumption per capita growth and the real riskless asset, a value for each state of nature, by matching their first two moments; namely mean and variance. Then I do the same for price dividend ratio matching the first two moments of market returns. Market real returns can be easily recovered using equation 2.10. For more details, please refer to Melino and Yang (2003).

Assuming consumption growth is a sufficient statistic for asset returns, then the first two moments of asset returns are enough to solve for the points of support for the risk free rate and equity processes in the Mehra-Prescott economy; see Epstein and Melino (1995). The model matches the first two moments of each variable as well as the real consumption per capita growth serial correlation.

Then, I calibrate the theoretical model, solving the system of Euler equations (eq. 2.6 and 2.7) for relative risk aversion and elasticity of intertemporal substitution coefficients. In solving the system of Euler equations initial values are chosen carefully. I look for values of relative risk aversion and elasticity of intertemporal substitution which best satisfy the system of these two equations.

Melino and Yang considered different cases for the state dependent parameters. First, they allowed only relative risk aversion to vary across states of nature. Then, they studied the model with only state dependent elasticity of intertemporal substitution. Finally, they considered the case when both relative risk aversion and elasticity of intertemporal substitution coefficients vary across states. Moreover, they studied the effect of a state dependent subjective discount factor in each of the previously mentioned cases. They concluded time varying subjective discount rates does not affect much the results. In addition, they emphasized a counter cyclical risk aversion plays an important role in explaining the equity premium data, but only when combined with modest cyclical variation in intertemporal substitution.

Motivated by their conclusions, I keep the subjective discount rate fixed. Risk aversion and elasticity of intertemporal substitution vary across states. I look for values of both parameters which solve the system of Euler equations; equations 2.6 and 2.7, without imposing any restrictions on the parameters. I assume

implicitly the elasticity of intertemporal substitution should take values less than one. I did not impose any restriction on the relative risk aversion coefficient or the value it can take. Despite attention has always been for risk averse agent, risk lovers exist. Recently, Crainich et al. (2013) supported the existence of mixed risk aversion and mixed risk lovers as well.

3.2 Data

I study the historical annual time series of the United States. Then I examine the historical annual series of United Kingdom, France, Germany, Italy, Japan and Canada. Each of these countries alone represents more than one percentage of the capitalized global equity market. Together they constitute seventy four percent on average of the capitalized global equity value throughout the period 1988-2012[6]. In addition, I study Italy because it is an example of a market where government's Treasury Bill has a higher real return on average than the stock market index. I consider the annual frequency because consumption for non durables and services as well as population, are available in annual frequency for all investigated countries.

I choose market indices which are national indices or can act as a representative to its stock market. Standard and Poor's 500 composite price index; hereafter S&P 500, represents the United States stock market index. While, FTSE 100 index, Nikkei 225, DAX, CAC40, FTSE All-World Europe index[7] and S&P/TSX Composite act as a benchmark for the stock market index of UK, Japan, Germany, France, Italy and Canada respectively.

The length of the time period studied for each country depends on data availability. I start from the first year on which data on all variables are available. I reexamine the United States original sample starting from 1890 until 1978 for comparability of results. 1890-1978 was the sample originally examined in Mehra and Prescott (1985) as well as in Melino and Yang (2003).Then I examine data on UK during the time period 1988-2012, Japan from 2004 until 2012, Germany during 1995-2012, France for the period 1990-2012, Italy over the time period 1999-2012, Canada from 1988-2012 and the United States during the time span from 1890 to 2012. The United States has the longest series of 123 years.

Throughout the paper, variables are in real values and with annual frequency. Variables used are as follows:

(a) A real risk free rate: it refers to the real yield on a relatively riskless short term security. Prior to 1920, the proxy for the risk free security in the United States was two-months to three-months Prime Commercial Paper. While form 1920-1930 Treasury Certificates represented the riskless security. For the rest of the United States sample, the three-month Treasury Bill rate was used instead. For all other countries, the risk free rate security is represented by each government's three month Treasury Bill. United States data are the one examined throughout the literature. Besides, Cochrane and Hansen (1992) emphasized that attempts to resolve the puzzle by accounting for mispricing of T-Bills are not likely to be productive.

(b) Real return of stock market index: it serves as a proxy for the return of a risky asset representing the market. All indices' returns are adjusted for their dividends. I calculate S&P 500 and S&P/TSX Composite returns using dividends, while for other indices; I use the total return index instead, because time series on dividends are missing.

(c) Real consumption per capita growth: the growth rate of real per capita consumption on non durables and services in each country's local currency.

(d) Real equity premium: is the excess real return of the stock market index over the real risk free rate. Real equity premium is calculated using each country's local currency data series.

(e) Consumer Price index: these series are used to calculate the consumer price index growth rate as a proxy for inflation rate. Then using the inflation rates or the consumer price index, one can get the real values of the aforementioned economic series. Consumer price indices used are with base year 2005.

Data on the United States before 1969 is from the website of Prof. Mehra. Consumer price index data is from the World Bank database with a base year 2005. While data for the consumption per capita on non durables and services for the United States is from the Bureau of Economic Analysis "BEA". For all the rest, data source is Datastream.

4. Empirical Results

Before proceeding to empirics, I mention how states are defined and I discuss the choice of the subjective discount factor value. Using a simple threshold model, the low state realizes when consumption per capita growth at time $t - 1$ are less than a value of one for the United States, Canada as well as France and less than a value of zero for the rest of the sample.

The subjective discount factor β can take values between zero and one. Barone-Adesi, Mancini and Shefrin (2013) restricted attention to values of subjective discount factor between 0.9 and 1.1, to address the aggregation bias in models with heterogeneous beliefs. In addition, they emphasized the time series for unconditional time preference tends to lie between 0.99 and 1.04. While, Melino and Yang (2003) found for values of $\beta \geq 0.97$ both elasticity of intertemporal substitution parameters fell in the admissible range; having values less than one. Increasing the subjective discount factor β had almost no effect on the risk aversion coefficient needed to rationalize the first two moments of asset returns but it decreased the value of the elasticity of intertemporal substitution. Given Melino and Yang results, I choose value for β in the range $0.97 \leq \beta < 1$. Hence, I keep the subjective discount factor constant, setting its value equal to 0.98. In section 4.3, I try other values for β as robustness check.

4.1 Re-examining the original sample: United States (1890-1978)

In this subsection, I reexamine the United States time series during the period extending from 1890 to 1978. It is the same sample of Melino and Yang (2003). In this time period the United States witnessed an average annual real rate of return on short term bills of 0.8% with a standard deviation of 0.056. While the average annual real rate of return on S&P 500 was 6.98% with a standard deviation of 0.165. This gives rise to an average equity premium of 618 basis points. Average gross real consumption growth was 1.018 with a standard deviation of 0.036 and autocorrelation of -0.14. The low state is defined when the real consumption per capita growth at time $t - 1$ is less than one. The transition matrix is shown below:

$$\prod = \begin{bmatrix} 0.43 & 0.57 \\ 0.57 & 0.43 \end{bmatrix}$$

Consequently, real gross per capita consumption growth's value in the low state is 0.98 and 1.054 in the high state. While real gross risk free rate in the low state equals to 0.95 and 1.064 in the high state. The price dividend ratio has a value of 23.5 in the low state and 27.8 in the high one.

There is a significant variation in real return across states. Real return depends on both current as well as next period's state. When current state is the low one, return varies between 2.4% and almost 30% depending on next period's state whether it will be a low or a high one respectively. While in the other case, having the high state as the current state of nature, real return can be negative; -0.1% when next period state is low or 9% when next period state is high. Up to here, the results are the same as Melino and Yang (2003).

The model generates real equity premium which varies significantly across states as well. With a low current state, equity premium is 7.3% when next period's state is low while it is 34.5% for a high upcoming state of nature. There is a notable difference between both values. In contrast, when the current state is the high one, equity premium is even negative with a value of 20% for a next period low state of nature. With both current and next period high state, the equity premium is almost 3%. Hence, the real equity premium is sensitive to current as well as next period's state of nature.

Given the values of real consumption per capita growth rate, real risk free rate and real returns in each state and setting $\beta = 0.98$, I solve the Euler equations for the state dependent $\alpha(s_t)$ and $\rho(s_t)$, representing the risk aversion and elasticity of intertemporal substitution respectively. Table 1 reports the results for both parameters in the subsample.

Table 1. Risk aversion and Elasticity of intertemporal substitution: United States (1890-1978)

	Low state	High state
Risk Aversion	47.6	-12
Elasticity of Intertemporal Substitution	0.03	0.004

Both coefficients differ significantly across states. The representative agent's low state relative risk aversion approximately equals 48, indicating a high level of risk aversion. In contrast, the agent is a risk lover in the high state with a value of risk aversion equal to -12. Therefore, the average relative risk aversion for the representative agent is 18. Hence, on average the representative agent is risk averse.

I found the elasticity of intertemporal substitution is countercyclical and close to zero. The intuition of a countercyclical elasticity of intertemporal substitution might be the following; in high states of nature the consumption growth rate is inelastic to variations in interest rate because the agent is already in a high state where his economic conditions are fine. While in the low state of nature, it can be more responsive to variations in interest rate due to the bad conditions, even though the difference between both values is small. It is familiar in asset pricing theory that individuals value more assets which pay in bad states. This might lead to having a lower elasticity of intertemporal substitution for a higher state of nature.

Using relative risk aversion and elasticity of intertemporal substitution coefficients, one can calculate the model's stochastic discount factor values. Table 2 reports those values. I find the model's stochastic discount factor varies significantly across states which increases its variability as desirable.

Table 2. State dependent stochastic discount factor: United States (1890-1978)

t+1 t	Low state	High state
Low state	2.35	0.0011
High state	0.2911	1.8132

According to the model, when the economy goes from a low to a high state of nature, the stochastic discount factor is very close to zero. In contrast, the representative agent values more assets which pay in the low state, given current state is low, leading to a high value for the stochastic discount factor in this case. These results imply the representative agent is extremely risk averse when current state is the low one. The situation is reversed for a high current state which indicates the representative agent is of a risk lover attitude. Most importantly, the model's stochastic discount factor values satisfy Hansen-Jagannathan bound.

The aforementioned results might seem unfamiliar because literature emphasizes the representative agent is always risk averse. Thus, I recover the implied stochastic discount factor from data and analyze how close it is to the model's values previously presented. It can be calculated without specifying any functional form for the stochastic discount factor, just satisfying the Euler equations given the returns and transition matrix values. Table 3 presents the implied values of the stochastic discount factor in this subsample.

Table 3. Implied stochastic discount factor (1890-1978)

t+1 t	Low state	High state
Low state	3.1134	-0.502
High state	0.2011	1.9192

The implied values vary significantly across states. The implied stochastic discount factor has a high value equal to 3 when both current and next states are low. Its value becomes negative when current state is low and next period's state is a high one, with a value of -0.5. The implied stochastic discount factor has a low value of 0.2 with a current high state and a low upcoming state. And its value becomes almost 2 when both next and current states are high. Implied values show the need for high variation in any model's stochastic discount factor in order to match data. Its values should differ significantly depending on both current and upcoming state of nature realization. It supports the state dependent preferences' model by showing the agent should be of a mixed risk averse type; i.e. he should be risk averse in the low current state while he should behave as a risk lover for a current high state of nature.

Nevertheless, these results can indicate the existence of an arbitrage opportunity, on real ground, going from a low to a high state of nature. But on average there are no arbitrage opportunities. A rational agent is going to exploit any arbitrage opportunity immediately which is the case in efficient markets. The existence of a stochastic discount factor in itself, assure the law of one price is satisfied by Riesz representation theorem.

The model's state dependent stochastic discount factor values are all strictly positive, as table 2 reported. In asset pricing models, the stochastic discount factor is nonnegative by construction. Knowing the data imply a negative value for the stochastic discount factor going from a low to a high state, it shows a weak

point in the functional form of asset pricing models' stochastic discount factor, which can't capture the negative values, when there is any. The main conclusion of a stochastic discount factor which varies significantly across states of nature and its dependence on both current as well as next period's state of nature is still valid though.

4.2 Robustness of results: State dependent preferences in selected developed countries

I investigate the model for UK, Japan, Germany, France, Italy and Canada. Besides, I extend the United States sample to check if there are any variations compared with the original sample (1890-1978) results. Many economic and financial events have happened during the period 1978-2012, which are worth including in the sample; the most recent is the financial crisis in 2008. I highlight all the results for Japan are depending on eight observations only (constrained by data availability). Table 4 reports summary statistics on the first two moments of each variable in the above-mentioned countries.

Table 4. First two moments' summary statistics

	Mean	Standard Deviation
Real Consumption per capita growth		
United States (1890-2012)	1.017	0.03
United Kingdom(1988-2012)	1.013	0.02
Japan (2004-2012)	1.004	0.01
Germany (1995-2012)	1.001	0.02
France (1990-2012)	1.006	0.01
Italy (1999-2012)	0.99	0.02
Canada (1988-2012)	1.005	0.01
Real Risk free rate		
United States (1890-2012)	1.012	0.05
United Kingdom(1988-2012)	1.031	0.03
Japan (2004-2012)	1.002	0.01
Germany (1995-2012)	1.012	0.02
France (1990-2012)	0.985	0.01
Italy (1999-2012)	1.046	0.03
Canada (1988-2012)	1.025	0.03
Real Returns		
United States (1890-2012)	1.077	0.17
United Kingdom(1988-2012)	1.077	0.17
Japan (2004-2012)	1.043	0.24
Germany (1995-2012)	1.075	0.26
France (1990-2012)	1.073	0.24
Italy (1999-2012)	1.04	0.24
Canada (1988-2012)	1.07	0.17

Satisfying these first two moments, the real gross per capita consumption growth's value in the low state is 0.99 except for Germany and Italy where it takes a value of 0.98 and 0.97 respectively. While it ranges between 1.048 for the United States and 1.014 for Italy in the high state of nature. On the other hand, real gross risk free rate in the low state lies between 0.94 in the United States and 1.016 in Italy. Rather, it takes values between 0.99 for France and 1.082 for the United States in the high state of nature. It is worthy to mention, real risk free rate has positive values in both high and low states for Italy and United Kingdom, while both values are negative in France. In fact, France has a negative risk free rate on average for the sample covered whereas Italy has the highest average gross value of 1.046.

Real returns are highly dependent on both current and next period's state. There are two common trends across countries. Going from a high to a low state of nature, real returns are negative. On the contrary, real returns are high varying between 31.7% in the United States and 47.5% in Germany for a current low state and high upcoming state. Notably, the Italian market is the only market for which real returns are negative when both current and next period's states are low.

Real equity premium values vary significantly across states as well. In all investigated countries, except for Italy, real equity premium is only negative going from a high to a low state of nature. Contrarily, the Italian real equity premium is only positive going from a low to a high state. With a low current state and high next period's state, markets experience high real equity premium ranging between 47.9% in Germany and 33.3% in Italy. On the other hand, real equity premium is highly negative in all markets when current state is high and upcoming state is low fluctuating, in absolute terms, between 31.7% in Italy and 21.1% in France.

Solving the model, I found relative risk aversion and elasticity of intertemporal substitution coefficients differ significantly across states. Table 5 presents the model's results for both coefficients. Results show the representative agent is extremely risk averse in the low state of nature; however, he is a risk lover in the high state. This kind of behavior is referred to as mixed risk aversion. On average, the representative agent is risk averse except for Italy where he is a risk lover.

Whereas the elasticity of intertemporal substitution varies significantly across states except for Japan. Its coefficient has a lower value in the high state than in the low state of nature. The intertemporal elasticity of substitution is countercyclical and close to zero.

Table 5. Model's results: Relative risk aversion and elasticity of intertemporal substitution

	Low state	High state	Average
Relative Risk Aversion			
United States (1890-2012)	53	-14.7	19.2
United Kingdom(1988-2012)	60.9	-10.9	25
Japan (2004-2012)	92.5	-36	28.3
Germany (1995-2012)	32.8	-24.6	4.1
France (1990-2012)	143	-33.9	54.6
Italy (1999-2012)	32	-70	-19
Canada (1988-2012)	39.5	-16.5	11.5
Elasticity of Intertemporal Substitution	**Low**	**High**	
United States (1890-2012)	0.047	0.006	
United Kingdom(1988-2012)	0.045	0.00001	
Japan (2004-2012)	0.3	0.286	
Germany (1995-2012)	0.01	0	
France (1990-2012)	0.02	0.014	
Italy (1999-2012)	0.014	0.0004	
Canada (1988-2012)	0.04	0	

With values for risk aversion as well as elasticity of intertemporal substitution, one can calculate the model's stochastic discount factor values. As can be seen from table 6, stochastic discount factor values are non negative in every state of nature, because as expected, the stochastic discount factor is non-negative by construction. The model shows, going from a low to a high state, the stochastic discount factor is close to zero for all examined countries. The model's stochastic discount factor has its highest values when both states; current and upcoming, are low or when both are high. Given its high volatility, the stochastic discount factor satisfies Hansen-Jagannathan bound.

Table 6. Stochastic Discount Factor: model's & implied values

SDF(t)	SDF(t+1)	Low state	High state
United States	Low	2.24 (3.13)	0.0001 (-0.64)
	High	0.12 (0.08)	1.9 (1.96)
United Kingdom	Low	1.5 (1.8)	0 (-0.4)
	High	0.2 (0.18)	1.3 (1.36)
Japan	Low	1.67 (1.76)	0.06 (-0.04)
	High	0.29 (0.09)	2 (2.24)
Germany	Low	1.7 (2.1)	0.15 (-0.17)
	High	0.4 (0.19)	1.5 (1.7)
France	Low	2.48 (3)	0.003 (-0.3)
	High	0.5 (0.4)	1.8 (1.96)
Italy	Low	2.6 (2.6)	0.094 (0.096)
	High	0.004 (-0.2)	2.6 (3)
Canada	Low	1.41 (1.68)	0.007 (-0.48)
	High	0.17 (0.169)	1.3 (1.3)

Implied stochastic discount factor values are between brackets.

Similarly, the implied stochastic discount factor varies significantly across states. It has a negative value when current state is low and next period's state is high for all the sample except for Italy where it takes a negative value going from a high to a low state of nature. Results show the stochastic discount factor values are sensitive not only to the current state of nature, but also to the next period's one. Its values differ significantly across states. The model resulted in stochastic discount factor values which are close enough to its implied values.

4.3 Robustness of results: subjective discount factor

I calibrated the model setting the subjective discount factor equal to 0.97 and 0.99. On one hand, changing the value of the subjective discount factor has negligible effect on the magnitude of average relative risk aversion except for Japan, Germany and Italy. For Italy and Germany, the difference in average relative risk aversion when $\beta = 0.97$ and $\beta = 0.98$ is negligible. With $\beta = 0.99$, average relative risk aversion increases significantly. Average relative risk aversion is sensitive to the subjective discount factor value in Japan. Increasing β lowers the average relative risk aversion. On the other hand, increasing the subjective discount factor β has no or negligible effect on the elasticity of substitution. When the effect is negligible, Increasing β decreases the elasticity of intertemporal substitution except for Italy, in which setting $\beta = 0.99$ increases the elasticity of intertemporal substitution.

However for all values of β, results support the representative agent is of a mixed risk averse type. Relative risk aversion and elasticity of intertemporal substitution being countercyclical is robust to changing the value of the subjective discount factor. $\beta = 0.97$ and $\beta = 0.99$ could not provide a lower norm for the Euler equations than when β was set equal to 0.98.

5. Conclusion

This paper studies the equity premium puzzle using state dependent recursive preferences in a Mehra-Prescott environment. State dependent preferences reflect the parameters' dependence on the state of nature being realized as well as the stochastic discount factor's dependence on both current and upcoming states. Applying the model to selected developed countries, I found a mixed risk averse representative agent could explain historical annual data. Relative risk aversion and elasticity of intertemporal substitution are countercyclical. The model's stochastic discount factor values vary significantly across states. Most importantly, its values satisfy Hansen-Jagannathan bound.

Meanwhile, the elasticity of intertemporal substitution coefficient is highly dependent on the state of nature, with only one exception for Japan. The model gave rise to low values of elasticity of intertemporal substitution, i.e. close to zero. Empirical studies using aggregate consumption data typically find elasticity

of intertemporal substitution value is close to zero; see Hall (1988). He supported the strong conclusion that elasticity of intertemporal substitution is unlikely to be much above 0.1, arguing consumption growth is completely insensitive to changes in interest rates in real world and, hence, intertemporal elasticity should be very close to zero and may well be zero[8]. The subsequent empirical macro literature (using aggregate data), by and large, has confirmed Hall's findings and provided further support for his conclusion; please refer to Campbell and Mankiw (1989) and Patterson and Pesaran (1992)[9]. Indeed, the smoothness of consumption on real grounds; being the core idea, in the face of real interest rate variation, suggests a low elasticity of intertemporal substitution.

Figure 1 plots historical[10] real equity premium series for the United States, being the longest time series available. Similar graphs for other countries are provided in the annex. A mixed risk averse representative agent supports the idea of having a real equity premium series fluctuating around the origin. In this sense, the real equity premium can have both negative and positive values.

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[8] Macroeconomic reasoning suggests a large value of elasticity of intertemporal substitution (close to one) being more consistent with observed aggregate behavior. In contrast, the more recent empirical consumption literature finds virtually no (or a very weak) response of consumption growth to expected interest rate movements suggesting an elasticity close to zero.
[9] Although this result has been challenged by a number of studies using micro data, the verdict for many seems to be that the average elasticity of substitution is close to zero. Numerous papers have estimated the elasticity of intertemporal substitution with United States data; e.g. Hansen and Singleton (1983), Hall (1988), and Campbell and Mankiw (1989). Campbell (2003) estimated it for international data. Moreover, Yogo (2004) estimated the Elasticity of intertemporal substitution in eleven developed countries. He found its value less than one and not significantly different from zero.
[10] There are three basic approaches for estimating equity premiums; surveys, implied premiums and historical premiums. The most widely used approach is using the historical values.

Figure(1): United States Real equity premium (1890-2012)
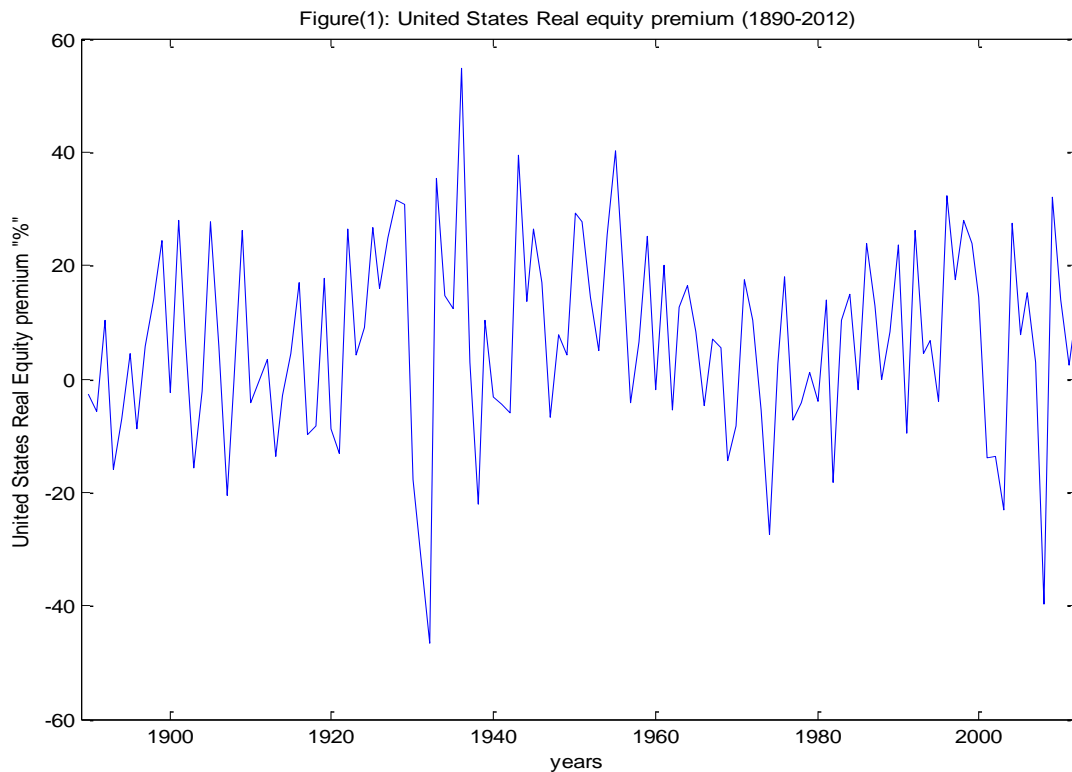
In Mehra and Prescott's model, with Constant Relative Risk Aversion utility function, the representative agent has only one relative risk aversion coefficient to calibrate the data in all states of nature, thus their model has no solution. Although in other settings, a high relative risk aversion coefficient of magnitude of approximately 50 is needed to explain the equity premium in the period 1890-1978. Regardless of the magnitude, a strictly positive risk aversion coefficient suggests the equity premium should be always positive. It fails in describing the dynamics of the equity premium series.

Melino and Yang's model allows the representative agent to have two values for the risk aversion coefficient; one value for each state of nature to be realized. It enables to discriminate representative agent's behavior in different states of nature. Calibrating their model, average risk aversion coefficient is positive, for all investigated countries except Italy where the representative agent is on average a risk lover. This explains the positive average real equity premium realized in all countries except for Italy where real equity premium is -0.7% on average. The level of average relative risk aversion differs across countries. It varies between -19 in Italy and 54.6 in France. Average risk aversion for the representative agent in Germany is 4.1, which lies within what most economists consider as acceptable level for risk aversion.

Differently from literature, I found the representative agent is risk averse in low states of nature while he is risk lover in the high ones for all examined countries; i.e. he has mixed risk aversion. It is the first time to introduce mixed risk aversion concept in the equity premium puzzle literature. Nevertheless, the level of the average risk aversion coefficient is still above what most economists believe acceptable except for Germany. However, Meyer and Meyer (2005) emphasized relative risk aversion for consumption, when broadly defined, can reach a level of twenty or even fifty for consumption levels below the mean.

Even though, with recursive preferences and state dependent preferences, having a representative agent with mixed risk aversion can explain the fluctuations in the real equity premium time series satisfying the first moments of real economic variables as well as the real consumption growth autocorrelation. Moreover, the model's stochastic discount factor values are very close to its implied values by data. The stochastic discount factor varies significantly across states. It depends on both current as well as next period's state of nature. Above all, its values satisfy Hansen-Jagannathan bound, which was another issue to match in the literature.

An additional interesting contribution of this paper is calculating the implied values of the real stochastic discount factor. Data imply a negative value for the stochastic discount factor going from a low to a high state for all investigated countries except Italy where the stochastic discount factor take a negative value going from a high to a low state. Knowing this, it shows a weak point in the asset pricing models' stochastic discount factor functional form, which never captures the negative values; when there is any. In contrast, the main conclusion of a stochastic discount factor which varies significantly across states as well as its dependence on both current and next period's state of nature is still valid.

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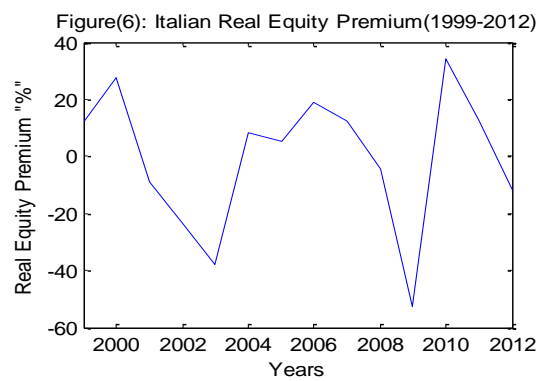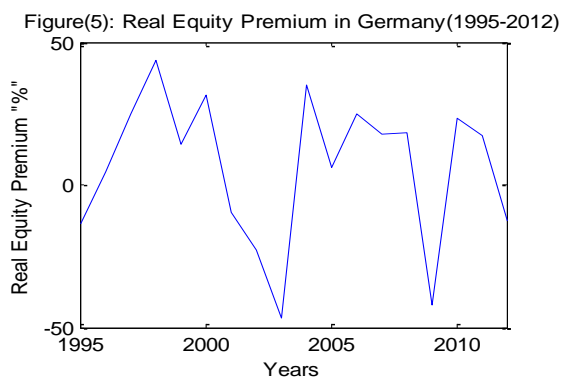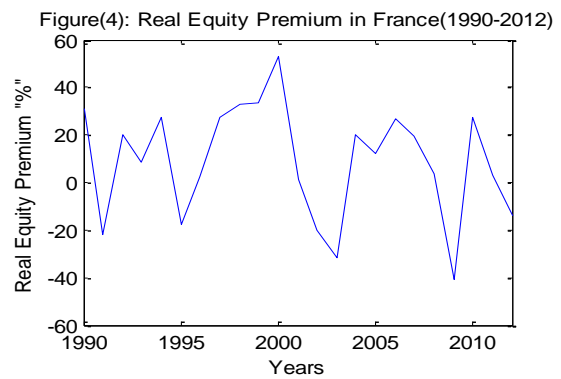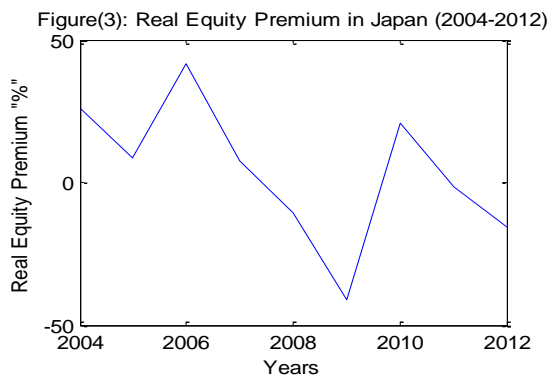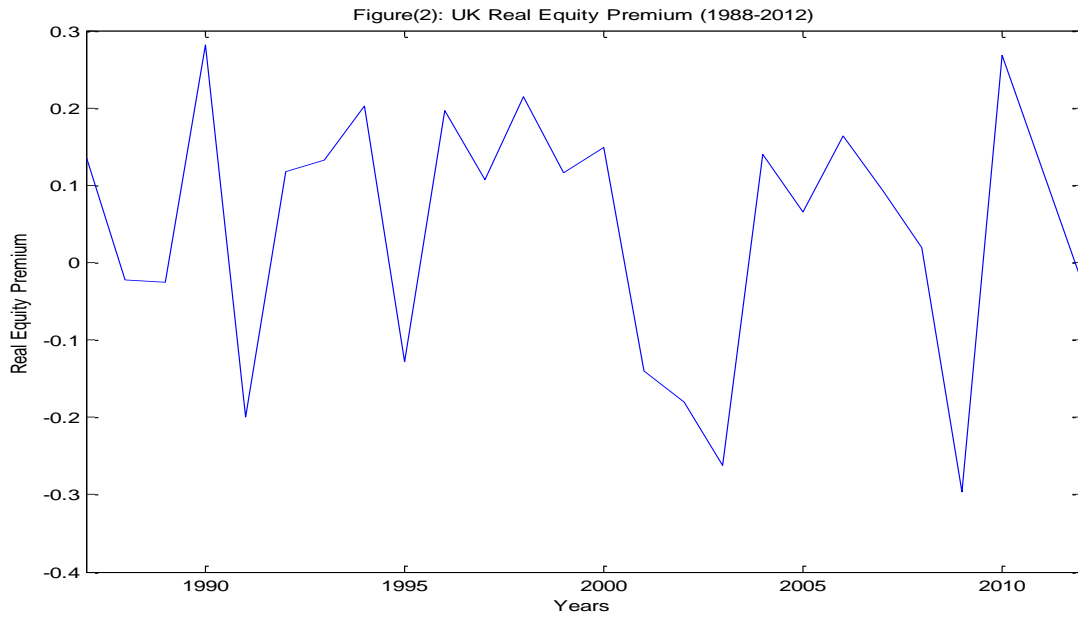
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Annex

Real Equity Premium graphs

Figure(2): UK Real Equity Premium (1988-2012)

Figure(3): Real Equity Premium in Japan (2004-2012)

Figure(4): Real Equity Premium in France(1990-2012)

Figure(5): Real Equity Premium in Germany(1995-2012)

Figure(6): Italian Real Equity Premium(1999-2012)

Figure(7): Canadian Real Equity Premium (1988-2012)